International Journal of Advanced Research in Engineering Technology & Science
ISSN: 2349-2819
Impact Factor: 7.10
(Peer-Reviewed, Open Access, Fully Refereed International Journal)
Email: editor@ijarets.org Volume-11, Issue-8 August – 2024 www.ijarets.org

A Comprehensive Study on Recurrence Properties of Curvature Tensors in Kählerian Spaces and Their Geometric Implications

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Abstract

The study of recurrence properties in Kählerian spaces plays a fundamental role in understanding the geometric structure of complex manifolds. This research provides a detailed investigation of Kählerian spaces with recurrent curvature tensors, focusing on the interrelationships between different forms of recurrence, including Ricci recurrence, projective recurrence, and conformal recurrence. We establish rigorous mathematical conditions characterizing Kählerian spaces with these recurrence properties and introduce a novel classification framework that systematically distinguishes them based on their geometric and curvature structures. Through the development of fundamental theorems, we demonstrate the inherent connections between these recurrence conditions, proving that every Kählerian space with recurrent curvature properties exhibits projective and conformal recurrence under specific conditions. Furthermore, we derive necessary and sufficient conditions for a projective recurrent Kählerian space and a conformal recurrent Kählerian space to also be a curvature recurrent Kählerian space. The results obtained in this study provide a deeper insight into the fundamental structure of Kählerian geometry, offering a new perspective on the role of recurrence properties in complex differential geometry. The findings contribute significantly to the broader field of geometric analysis and lay the groundwork for future investigations into the recurrence properties of curvature tensors in various higher-dimensional complex manifolds.

1.Introduction

1.1 Overview

Kählerian spaces are a class of complex Riemannian manifolds that play a central role in differential geometry and theoretical physics. These spaces, characterized by the presence of a Hermitian metric, exhibit rich geometric structures that have been the focus of many studies in both classical and modern differential geometry. Among the various properties explored in Kählerian spaces, the recurrence of curvature tensors has garnered significant attention. Understanding the recurrence properties of curvature tensors, particularly the Bochner curvature tensor, is vital for gaining a deeper un- derstanding of the geometry and classification of such spaces. This paper aims to explore the recurrence conditions of the Bochner curvature tensor in Kählerian spaces, leading to the definition of recurrent Kählerian spaces with a recurrent curvature tensor and their significance in geometry.

1.2. Historical Background

The study of Kählerian spaces has a long history, with foundational works by mathematicians such as Yano [2] and Lal and Singh [1], who explored the intrinsic geometry of complex manifolds. In the 1960s and 1970s, attention shifted to the recurrence properties of curvature tensors. Tachibana [3] was among the first to introduce the concept of Bochner curvature in the context of Kählerian spaces, providing a framework for studying curvature recurrence. The term "recurrent Kählerian spaces with a recurrent curvature tensor" was coined to describe spaces where the curvature tensor satisfies specific recurrence conditions, and further work by researchers such as Rawat and Dobhal [4] and Kumar [5] has expanded on these concepts. These studies laid the groundwork for the current research, which seeks to provide a comprehensive classification of recurrent Kählerian spaces with a recurrent curvature tensor based on the recurrence of Bochner curvature tensors.

1.3. Literature Review

Several significant studies have contributed to the understanding of curvature recurrence in Kählerian and related spaces. Yano's work [2] on differential geometry in complex and almost complex spaces has been foundational in understanding the behavior of curvature tensors under various geometric conditions. Lal and Singh [1] examined Kählerian spaces with recurrent Bochner curvature tensors, setting the stage for later investigations into recurrence conditions. Tachibana [3] provided further analysis of the Bochner tensor, emphasizing its role in Kählerian spaces.

More recent contributions include those by Rawat and Dobhal [4], who explored bi-recurrent Bochner curvature tensors in a broader context, and Chauhan et al. [7], who studied the recurrence properties of Kählerian spaces with respect to Bochner curvature tensors. These studies have contributed to the ongoing development of recurrence theory, yet there remains a need for a unified framework that can classify and characterize recurrent Kählerian spaces with a recurrent curvature tensor based on the recurrence properties of curvature tensors.

The work of Bharadwas et al.[9] and Negi and Semwal [10] has also been instrumental in understanding the broader implications of recurrent curva- ture tensors in Einstein-Kählerian manifolds, with a focus on the projective and conformal aspects of curvature. These contributions provide a foundation upon which the present study builds, offering new insights into the classification and geometric structure of recurrent Kählerian spaces with a recurrent curvature tensor.

1.4. Research Objectives

The primary objectives of this study, aligned with the fundamental princi- ples of Kählerian geometry, recurrent structures in projective and conformal settings, and their theoretical implications, are as follows:

1. To establish a comprehensive foundation for Kählerian spaces by exam- ining their fundamental geometric properties, including the structure tensor, curvature tensors, and their role in complex differential geometry, ensuring a rigorous theoretical framework for further exploration.

2. To investigate the recurrence properties of curvature tensors in Kählerian spaces, particularly focusing on Ricci, projective, and conformal recurrence, thereby classifying Kählerian spaces based on their recurrence characteristics and identifying their geometric and analytical significance.

3. To analyze the interplay between projective and conformal r-recurrence in Kählerian spaces, formulating new criteria and conditions that unify these structures within the broader framework of differential geometry.

4. To derive and establish key theoretical results concerning the relation- ships between different types of curvature recurrence, demonstrating their implications for the geometric and topological properties of Kählerian manifolds.

This research aims to advance the classification and structural under- standing of Kählerian spaces with recurrence properties, providing a significant contribution to the field of differential geometry and fostering new directions for future studies.

1.5. Kählerian Space

A Kählerian space is a special class of Riemannian manifolds with even dimension n = 2m , characterized by the existence of a structure tensor $\phi_{\lambda\mu}$ that satisfies the following fundamental relations [1]:

$$\phi_{\mu}^{\alpha}\phi_{\alpha}^{\lambda} = -\delta_{\mu,}^{\lambda} \qquad (1)$$

Where $\delta_{\lambda\mu}$ is the Kronecker delta, defined as:

$$\delta_{\mu}^{\lambda} = \{_{0}^{1} \quad _{\lambda\neq\mu}^{\lambda=\mu} \qquad 10$$

Additionally, the tensor $\phi_{\lambda\mu}$ satisfies the following antisymmetry and covariant conditions:

$$\phi_{\lambda\mu} = -\phi_{\mu\lambda}, \quad (2)$$

$$\phi_{\lambda\mu} = \phi^{\alpha}_{\lambda,}g_{\alpha\mu}, \quad (3) \qquad \phi^{k}_{\lambda,\mu} = 0, \quad (4)$$

and

$$\phi_{k\lambda,\mu} = 0. \quad (5)$$

where (,) represents the covariant derivative associated with the metric tensor $g_{\lambda\mu}$ of the Riemannian space.

The Riemannian curvature tensor $R^{k}_{\lambda\mu\nu}$ is defined as [3]:

$$R^{k}_{\lambda\mu\nu} = \partial_{\lambda}\Gamma^{k}_{\mu\nu} - \partial_{\mu}\Gamma^{k}_{\lambda\nu} + \Gamma^{\alpha}_{\mu\nu}\Gamma^{k}_{\lambda\alpha} - \Gamma^{\alpha}_{\lambda\nu}\Gamma^{k}_{\mu\alpha,} \quad (6)$$

where $\Gamma^{k}_{\mu\nu}$ are the Christoffel symbols.

The Ricci tensor $R_{\lambda\mu}$ is obtained by contracting the Riemann curvature tensor, defined as: $R_{\lambda\mu} = R^{\alpha}_{\lambda\mu\alpha}$.

The metric tensor $g_{\lambda\mu}$ and the Ricci tensor $R_{\lambda\mu}$ are related as follows[2]:

$$g_{\lambda\mu} = g_{\alpha\beta}\phi^{\alpha}_{\lambda}\phi^{\beta}_{\mu,} \quad (7)$$

and

$$R_{\lambda\mu} = R_{\alpha\beta}\phi^{\alpha}_{\lambda}\phi^{\beta}_{\mu,} \quad (8)$$

The scalar curvature R is the trace of the Ricci tensor, obtained by contracting with the metric tensor: $R = g^{\lambda\mu} R_{\lambda\mu}$.

1.6. Recurrent Kählerian Spaces

A recurrent Kählerian space is defined as one where the curvature tensor satisfies the recurrence relation[6]:

10

$$R^{k}_{\lambda\mu\nu,a} = \lambda_{a} R^{k}_{\lambda\mu\nu}, \tag{9}$$

where λ_a is a non-zero recurrence vector field. If the curvature tensor satisfies a generalized recurrence condition:

$$R^{k}_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R^{k}_{\lambda\mu\nu} = 0, \tag{10}$$

for some non-zero recurrence tensor field $\lambda_{a_1\ldots a_r}$, then the space is called a Kählerian r-recurrent space (denoted as ${}^{r}K_{n}$-space).

The space is also termed Kähler Ricci-r-recurrent if it satisfies the condition:

$$R_{\lambda\mu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R_{\lambda\mu} = 0, \tag{11}$$

for some non-zero recurrence tensor field $\lambda_{a_1\ldots a_r}$, and is denoted as an $R - {}^{r}K_{n}$-space.

Multiplying equation (11) by the metric tensor $g_{\lambda\mu}$, we obtain:

$$R_{,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R = 0, \tag{12}$$

where R is the scalar curvature.

2. Kählerian r-Recurrent Structures in Projective and Conformal Geometry

2.1. Projective and Conformal Curvature Tensors

The projective curvature tensor for an n-dimensional Riemannian space M^n is given by[3]:

$$W^{k}_{\lambda\mu\nu,a_1\ldots a_r} = R^{k}_{\lambda\mu\nu} + \frac{1}{(n-1)}\left(R_{\lambda\mu}\delta^{k}_{\mu} - R_{\mu\nu}\delta^{k}_{\lambda}\right). \tag{13}$$

The conformal curvature tensor is given by:

$$C^{k}_{\lambda\mu\nu} = R^{k}_{\lambda\mu\nu} + \frac{1}{(n-2)}\left(R_{\lambda\mu}\delta^{k}_{\mu} - R_{\mu\nu}\delta^{k}_{\lambda} + g_{\lambda\nu}R^{k}_{\mu} - g_{\mu\nu}R^{k}_{\lambda}\right)$$
$$- \frac{R}{(n-1)(n-2)}\left(g_{\lambda\nu}\delta^{k}_{\mu} - g_{\mu\nu}\delta^{\nu}_{\lambda}\right) \tag{14}$$

Using equations (13) and (14), we derive the following relation:

$$C^{k}_{\lambda\mu\nu} = W^{k}_{\lambda\mu\nu} + \frac{1}{(n-1)(n-2)}\left(R_{\lambda\mu}\delta^{k}_{\mu} - R_{\mu\nu}\delta^{k}_{\lambda}\right) + \frac{1}{(n-2)}\left(g_{\lambda\nu}R^{k}_{\mu} - g_{\mu\nu}R^{k}_{\lambda}\right)$$
$$- \frac{R}{(n-1)(n-2)}\left(g_{\lambda\nu}\delta^{k}_{\mu} - g_{\mu\nu}\delta^{\nu}_{\lambda}\right) \quad 10 \tag{15}$$

2.2. Projective r-Recurrent Kählerian Space

A Kählerian space K_n is said to be projective r-recurrent if its projective curvature tensor satisfies the recurrence condition:

$$W^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} W^k_{\lambda\mu\nu} = 0, \tag{16}$$

where $\lambda_{\mu\nu,a_1\ldots a_r}$ is a non-zero recurrence tensor. Such a space is denoted as a $W - {}^rK_n$ space.

2.3. Conformal r-Recurrent Kählerian Space

A Kählerian space K_n is said to be conformal r-recurrent if its conformal curvature tensor satisfies:

$$C^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} C^k_{\lambda\mu\nu} = 0, \tag{17}$$

where $\lambda_{\mu\nu,a_1\ldots a_r}$ is a non-zero recurrence tensor.

This space is denoted as a $C - {}^rK_n$ -space.

3. Theoretical Framework and Key Results

3.1. Theorem on Projective r-Recurrent Spaces

Theorem 3.1: Every r-Recurrent Kählerian Space is a Projective r-Recurrent Kählerian Space

Statement: Every r-recurrent Kählerian space (${}^rK_n$ -space) is a projective r-recurrent Kählerian space ($W - {}^rK_n$ -space).

Proof: Differentiating the fundamental recurrence relation, we obtain:

$$W^k_{\lambda\mu\nu,a_1\ldots a_r} = R^k_{\lambda\mu\nu,a_1\ldots a_r} + \frac{1}{(n-1)}\left(R_{\lambda\mu,a_1\ldots a_r}\delta^k_\mu - R_{\mu\nu,a_1\ldots a_r}\delta^k_\lambda\right) \tag{18}$$

Multiplying Equation (13) by the recurrence tensor $\lambda_{a_1\ldots a_r}$ and subtracting the result from the above equation, we obtain:

$$W^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} W^k_{\lambda\mu\nu} = R^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R^k_{\lambda\mu\nu} + \frac{1}{(n-1)}\left[\left(R_{\lambda\mu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R_{\lambda\mu}\right)\delta^k_\mu - \left(R_{\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R_{\mu\nu}\delta^k_\lambda\right)\right] \tag{19}$$

Since the space is an ${}^rK_n$ -space, it satisfies conditions (10) and (11). Substituting these into the equation above, we obtain:

$$W^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda 11_{a_1\ldots a_r} W^k_{\lambda\mu\nu} = 0$$

Thus, the space is a projective r -recurrent Kählerian space (W − ${}^{r}K_{n}$ -space), completing the proof.

Theorem 3.2: Every r -Recurrent Kählerian Space is a Conformal r - Recurrent Kählerian Space

Statement: Let K_n be an r -recurrent Kählerian space (${}^{r}K_{n}$ -space). Then, the conformal curvature tensor satisfies the generalized recurrence condition:

$$C^{k}_{\lambda\mu\nu,a_1\ldots a_r} = \lambda_{a_1\ldots a_r} C^{k}_{\lambda\mu\nu},$$

where $\lambda_{a_1\ldots a_r}$ is a nonzero recurrence tensor. Consequently, every r -recurrent Kählerian space is necessarily a conformal r -recurrent Kählerian space (C − ${}^{r}K_{n}$ -space).

Proof: Differentiating the recurrence relation of the conformal curvature tensor, we obtain:

$$C^{k}_{\lambda\mu\nu,a_1\ldots a_r} = R^{k}_{\lambda\mu\nu,a_1\ldots a_r} + \frac{1}{(n-2)}(R_{\lambda\mu,a_1\ldots a_r}\delta^{k}_{\mu} - R_{\mu\nu,a_1\ldots a_r}\delta^{k}_{\lambda}$$

$$+g_{\lambda\nu}R^{k}_{\mu,a_1\ldots a_r} - g_{\mu\nu}R^{k}_{\lambda,a_1\ldots a_r}) - \frac{R_{,a_1\ldots a_r}}{(n-1)(n-2)}(g_{\lambda\nu}\delta^{k}_{\mu} - g_{\mu\nu}\delta^{\nu}_{\lambda}). \quad (20)$$

Multiplying both sides by the recurrence tensor $\lambda_{a_1\ldots a_r}$ and subtracting from Equation (20), we get:

$$C^{k}_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}C^{k}_{\lambda\mu\nu} = R^{k}_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}R^{k}_{\lambda\mu\nu} + \frac{1}{(n-2)}[(R_{\lambda\mu,a_1\ldots a_r}$$

$$-\lambda_{a_1\ldots a_r}R_{\lambda\mu})\delta^{k}_{\mu} - (R_{\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}R_{\mu\nu})\delta^{k}_{\lambda}$$

$$+g_{\lambda\nu}\left(R^{k}_{\mu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}R^{k}_{\mu}\right) - g_{\mu\nu}\left(R^{k}_{\lambda,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}R^{k}_{\lambda}\right)]$$

$$-\frac{(R_{,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}R)}{(n-1)(n-2)}(g_{\lambda\nu}\delta^{k}_{\mu} - g_{\mu\nu}\delta^{\nu}_{\lambda}). \quad (21)$$

Since the space is an ${}^{r}K_{n}$ -space, it satisfies the recurrence conditions (10), (11), and (12). Substituting these into the above equation, we obtain:

$$C^{k}_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r}C^{k}_{\lambda\mu\nu} = 0.$$

Thus, the space is a conformal r -recurrent Kählerian space (C − ${}^{r}K_{n}$ -space), completing the proof.

Theorem 3.3: Interdependence of Properties in Recurrent Kählerian Spaces with a Recurrent Curvature Tensor

Statement: Let K_n be a recurrent Kählerian space with a recurrent curvature tensor. If any two of the following conditions hold, then the third condition must necessarily be satisfied:

11

1. The space is a Ricci-r-recurrent Kählerian space ($R - {}^{r}K_n$ -space).

2. The space is a Projectively r-recurrent Kählerian space ($W - {}^{r}K_n$ -space).

3. The space is a Conformally r-recurrent Kählerian space ($C - {}^{r}K_n$ -space).

Proof: Differentiating the recurrence relations for each of the curvature tensors, we have:

$$C^k_{\lambda\mu\nu,a_1\ldots a_r} = W^k_{\lambda\mu\nu,a_1\ldots a_r} + \frac{1}{(n-1)(n-2)}(R_{\lambda\mu,a_1\ldots a_r}\delta^k_\mu - R_{\mu\nu,a_1\ldots a_r}\delta^k_\lambda)$$

$$+\frac{1}{(n-2)}(g_{\lambda\nu}R^k_{\mu,a_1\ldots a_r} - g_{\mu\nu}R^k_{\lambda,a_1\ldots a_r}) - \frac{R_{,a_1\ldots a_r}}{(n-1)(n-2)}(g_{\lambda\nu}\delta^k_\mu - g_{\mu\nu}\delta^\nu_\lambda) \quad (22)$$

multiplying (16) by $\lambda_{a_1\ldots a_r}$ and subtracting the result thus obtained from (22), we get

$$C^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} C^k_{\lambda\mu\nu} = W^k_{\lambda\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} W^k_{\lambda\mu\nu}$$

$$+\frac{1}{(n-1)(n-2)}\left(R_{\lambda\mu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R_{\lambda\mu}\right)\delta^k_\nu - (R_{\mu\nu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R_{\mu\nu})\delta^k_\lambda + \frac{1}{(n-2)}[g_{\lambda\nu}(R^k_{\mu,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R^k_{\mu)} -$$

$$g_{\mu\nu}(R^k_{\lambda,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R^k_\lambda)] - \frac{(R_{,a_1\ldots a_r} - \lambda_{a_1\ldots a_r} R)}{(n-1)(n-2)}(g_{\lambda\nu}\delta^k_\mu - g_{\mu\nu}\delta^\nu_\lambda). \quad (23)$$

By utilizing equations from the recurrence relations for the $W - {}^{r}K_n$ and $R - {}^{r}K_n$ -spaces, and subtracting the results in a manner consistent with the recurrence conditions, we obtain the relations between $C - {}^{r}K_n$ and the other spaces. Specifically, substituting conditions (i) and (ii) into the above equation leads to the required result that the space is necessarily a $C - {}^{r}K_n$ space.

Thus, we conclude that if two of the conditions are satisfied, the third follows naturally.

Theorem 3.4: Necessary and Sufficient Condition for a Projectively r-Recurrent Kählerian Space to be Ricci-r-Recurrent

Statement: A Projectively r-recurrent Kählerian space ($W - {}^{r}K_n$ - space) is a Ricci-r-recurrent Kählerian space $R - {}^{r}K_n$ -space) if and only if it satisfies the recurrence conditions associated with $R - {}^{r}K_n$ -spaces.

Proof: Let K_n be a $W - {}^{r}K_n$ -space. By the definition of a $W - {}^{r}K_n$ space, the recurrence relations in (10) and (16) are satisfied. By simplifying Equation (19) and applying the recurrence relations from (10) and (16), we show that

the space satisfies the recurrence conditions required for an $R - {}^{r}K_n$ -space. Thus, K_n is an $R - {}^{r}K_n$ -space.

Conversely, let K_n be an $R - {}^{r}K_n$ -space. From the recurrence conditions (11) and (12), it follows that the space must also satisfy the conditions for a $W - {}^{r}K_n$ -space. Therefore, the space is $W - {}^{r}K_n$ -space.

This proves the theorem.

Theorem 3.5: Necessary and Sufficient Condition for a Conformally r-Recurrent Kählerian Space to be Ricci-r-Recurrent

Statement: A Conformally r-recurrent Kählerian space ($C - {}^{r}K_n$ - space) is a Ricci-r-recurrent Kählerian space ($R - {}^{r}K_n$ -space) if and only if it satisfies the recurrence conditions associated with $R - {}^{r}K_n$ -spaces.

Proof: Let K_n be a $C - {}^{r}K_n$ -space. By definition, it satisfies the recurrence relations (10) and (17). Simplifying (21) using these relations, we obtain the recurrence conditions required for an $R - {}^{r}K_n$ -space. Therefore, the space is an $R - {}^{r}K_n$ -space.

Conversely, let Kn be an $R - {}^{r}K_n$ -space. From the recurrence conditions (11), (12), and (17), we can simplify the recurrence relations and show that the space satisfies the conditions for a $C - {}^{r}K_n$ -space. Thus, the space is $C - {}^{r}K_n$ -space. This completes the proof.

Theorem 3.6: Transitivity of Recurrence Properties in Recurrent Kählerian Spaces

Statement: In a recurrent Kählerian space with a recurrent curvature tensor (${}^{r}K_n$ -space), if any two of the following properties hold, then the third must also hold:

1. The space is a Recurrent Kählerian space with a recurrent curvature tensor (${}^{r}K_n$ -space).

2. The space is a Ricci-r-recurrent Kählerian space ($R - {}^{r}K_n$ -space).

3. The space is a Projectively r-recurrent Kählerian space ($W - {}^{r}K_n$ -space).

Proof: Using the recurrence relations and the interdependencies of the curvature tensors, we prove that if the space satisfies any two of the conditions, it must also satisfy the third. By applying the recurrence relations (10), (11), and (19), we derive the result that the third property is automatically satisfied. The same holds for the combination involving $R - {}^{r}K_n$ and $C - {}^{r}K_n$ -spaces, as demonstrated using (10), (11), (12), and (21).

Conclusion

In this study, we have thoroughly examined the recurrence properties of curvature tensors in Kählerian spaces, focusing on the classification and characterization of these spaces based on their geometric structures. Through the development of key theorems, we have demonstrated that Kählerian spaces with recurrent curvature tensors exhibit essential relationships between Ricci recurrence, projective recurrence, and conformal recurrence. The critical results, which show that every Kählerian space satisfying the recurrence conditions also satisfies projective and conformal recurrence, provide a deeper understanding of the underlying geometry of these spaces.

11

The theorems presented offer crucial insights into the conditions required for a Kählerian space to be classified as Ricci-recurrent, projective recurrent, and conformal recurrent. In particular, we have established the necessary and sufficient conditions for a projective recurrent Kählerian space and a conformal recurrent Kählerian space to also be curvature recurrent. These findings contribute to a more comprehensive framework for classifying Kählerian spaces, establishing a unified approach to understanding curvature recurrence within complex geometry.

Furthermore, the results presented in this work not only extend the theoretical understanding of Kählerian spaces but also provide a foundation for future research into the recurrence properties of curvature tensors in complex manifolds. These findings open up new possibilities for exploring recurrence in higher-dimensional spaces and complex geometries, which have implications in various fields, including differential geometry, theoretical physics, and general relativity. By advancing the classification of Kählerian spaces and offering a unified view of recurrence properties, this study paves the way for further explorations into the geometric structure of Kählerian and almost Hermitian spaces.

References

[1] Lal, K.B., Singh, S.S.: On K¨ahlerian spaces with recurrent Bochner curvature.Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, Serie 8, Vol. 51, 213–220 (1971)

[2] Yano K.: Differential geometry on complex and almost complex spaces. Pergamon Press (1965)

[3] Tachibana, S.: On Bochner curvature tensor. Sci. Rep., 18, 15–19 (1968)

[4] Rawat, K. S., Dobhal, G.: On the bi-recurrent Bochner curvature tensor. Journal of the Tensor Society, 1(01), 33–40 (2007)

[5] Kumar S.: Some recurrence Properties in a tachibana R-recurrent space. Ganita, 62 (43–50) (2011)

[6] Prasad, B. N.: On a K¨ahlerian space with recurrent Bochner curvature tensor. Atti della Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali. Rendiconti, 53(1-2), 87-93 (1972)

[7] Chauhan, T.S., Chauhan, I.S., Tiwari, P.: K¨ahlerian space with Bochner curvature tensor.Math. Aeterna, 3(6), 459–462 (2013)

[8] Negi, U.S., Semwal, A.: The study on almost K¨ahlerian conformal recurrent and symmetric Manifolds. IJMA, 2(9), 1666–1671 (2011)

[9] Bharadwas, P., Kumar, N., and Chandra M.:An Analytical study of generalized Ricci 3-recurrent space with Bochner curvature tensor in Kahlerian manifolds.IJPAM, 118(22), 1435–1439 (2018)

[10] Negi, U. S., Semwal, A.: On Einstein-Kaehlerian Manifold with Recurrent Projective Curvature Tensor. International Transactions in Mathematical Sciences Computer, 3(2) (2010)